LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2015**

MT 2503 - ANALY. GEOM. OF 3D, FOURIER SERIES & NUM. THEORY

Date : 17/04/2015 Dept. No. Max. : 100 Marks

Time : 01:00-04:00

PART – A

ANSWER ALL THE QUESTIONS: (10 x 2 = 20)

1. Find the equation to the plane through (1,2,3) parallel to the plane 4x+5y-3z+7 = 0.
2. Write the equation of the symmetric form of the straight line.
3. Find the centre and radius of the sphere $x^2+y^2+z^2-2y-4z = 11$.
4. Write the equation of the tangent plane to the sphere.
5. Write the formula for Fourier series.
6. Define odd and even functions.
7. Find the number of divisors of 360.
8. State Fermat's theorem.
9. If a, b, c are positive, and not all equal, then show that (a+b+c) (bc+ca+ab) > 9abc.
10. Show that $n^n > 1 .3 .5. \ldots . (2n-1)$.

PART – B

ANSWER ANY FIVE QUESTIONS (5 x 8 = 40)

11. Show that, if a plane has intercepts a, b,c on the coordinate axes and is at a distance p from the origin , then $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$.

12. Show that the lines $\frac{x+1}{-3}=\frac{y+10}{8}=\frac{z-1}{2}$; $\frac{x+3}{-4}=\frac{y+1}{7}=\frac{z-4}{1}$ are coplanar. Find their point of intersection and the equation to the plane containing them.

13. Find the equation of the sphere whose centre is the point (6,-1,2) and which touches the plane 2x – y + 2z - 2 = 0.

14. Find the equation of the sphere through the points (2,3,1), (5, -1, 2), (4, 3, -1) and (2, 5, 3).

15. Express $f(x) = x, \quad (-\pi < x < \pi)$ as a Fourier series with period 2π.

16. Find the highest power of 3 dividing 1000!.

17. Show that $13^{2n+1} + 9^{2n+1}$ is divisible by 22.

18. Show that $(x^m + y^m)^n < (x^n + y^n)^m$ if m > n.

PART – C

ANSWER ANY TWO QUESTIONS (2x 20 = 40)

19. (a) Find the equation of the plane through the points (2, 2, 1) and (0, 3, 6) and perpendicular to the plane 2x + 6y + 6z = 9.

(b)Find the shortest distance between the lines

$\frac{x-3}{-1}=\frac{y-4}{2}=\frac{z+2}{1}$ *and* $\frac{x-1}{1}=\frac{y+7}{3}=\frac{z+2}{2}$.

20. The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the axes in A, B, C. Find the equation of the circumcircle of the triangle ABC and determine also the coordinates of the centre and radius.

21. (a) Show that $x^2=\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}(-1)^n\frac{\cos nx}{n^2}$ in the interval $(-\pi\le x\le\pi)$.

(b) Show that 8^{th} power of any number is of the form 17m or $17m\pm1$.

22. (a) State and prove Wilson's theorem.

(b) If $s=a_1+a_2+...+a_n$ then show that $\frac{s}{s-a_1}+\frac{s}{s-a_2}+...+\frac{s}{s-a_n}>\frac{n^2}{n-1}$ unless $a_1=a_2=...=a_n$.

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